REAL ESTATE THEORY AND MODELLING

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CHRONICLE N°6

The Gordon-Shapiro model for real estate (1/4)

It is not uncommon to come across mathematical relationships derived from the Gordon-Shapiro model in a real estate article, without any mention being made of them. But what exactly is the Gordon-Shapiro model?

The Gordon-Shapiro model applied to real estate tells us that the initial income return (ir_1) is equal to the total return (tr) minus the growth rate of the net operating income (g). Or that the total return is equal to the initial income return plus the growth rate of the net operating income. Or, finally, that the initial theoretical price of the property is equal to the initial net operating income (noi_1) divided by the difference between the discount rate (r) (which is the same as the expected total return), minus the growth rate of the net operating income.

All this is based on two simplifying assumptions. The growth rate of the net operating income must be strictly constant over time and the calculation horizon, and therefore the investment horizon, must be infinite.

$$ir_1 = tr - g \Leftrightarrow tr = ir_1 + g \Leftrightarrow P_0 = \frac{noi_1}{r - g}$$

The Gordon-Shapiro model (1956)[1] was originally developed to estimate the price of a share. It can be easily adapted to the valuation of a real estate asset, its starting point being the discounted cash flow valuation method.

$$(1)\ P_0 = \frac{noi_1}{(1+r)} + \frac{P_1}{(1+r)}$$

with:

- P_0 : the price today
- P_1 : the price at the end of period 1
- noi_1 : the initial net operating income for period 1
- r : the discount rate

[1] Gordon M.J. et E. Shapiro, (1956) "Capital Equipment Analysis: The Required Rate of Profit", Management Science, 3, pp. 17–35.

And therefore, by definition:

$$(2)\ P_1 = \frac{noi_2}{(1+r)} + \frac{P_2}{(1+r)}$$

with:

P_1 : the price at the end of period 1
P_2 : the price at the end of period 2
noi_2 : the net operating income for period 2
r : le taux d'actualisation

If I replace (2) in (1) I get:

$$(3)\ P_0 = \frac{noi_1}{(1+r)} + \frac{}{(1+r)} . \left(\frac{noi_2}{(1+r)} + \frac{P_2}{(1+r)} \right)$$

$$(4)\ P_0 = \frac{noi_1}{(1+r)} + \frac{noi_2}{(1+r)^2} + \frac{P_2}{(1+r)^2}$$

The general formula is as follows if I continue to T periods:

$$(5)\ P_0 = \sum_{t=1}^{T} \frac{noi_t}{(1+r)^t} + \frac{P_T}{(1+r)^T}$$

And if I go on ad infinitum:

$$(6)\ P_0 = \frac{noi_1}{(1+r)} + \frac{noi_2}{(1+r)^2} + \cdots + \frac{noi_t}{(1+r)^t} + \cdots$$

This leads to the general formula:

$$(7)\ P_0 = \sum_{t=1}^{\infty} \frac{noi_t}{(1+r)^t}$$

Today's price is equal to the discounted sum of future revenues, the discounted cash-flows.

So far, nothing extraordinary for anyone with a little knowledge of financial mathematics. For the rest of us, you should know that this formula, which defines the price as the discounted sum of the future income from an asset or any other investment, is the cornerstone of modern financial analysis. As we generally do not know what the future income will be, it is most often presented as an "expectation". In other words, today's price should be equal to the discounted 'expectation' of future income, i.e. the most likely average value of future income.

This is partly, and only partly, one of the reasons why there are buyers and sellers on a market, because they do not share the same 'expectation' of future income and therefore do not have the same opinion, either on the 'right' price today, or on the long-term value of the property.

The simplifying assumption introduced by Gordon-Shapiro is that net operating income grows at a constant rate g. i.e. :

$$(8)\ noi_t = (1+g)^t . noi_0 = (1+g)^{t-1} . noi_1$$

with: g : the growth rate of the net operating income

Equation (7) can therefore be written as:

$$(9)\ P_0 = \sum_{t=1}^{\infty} \frac{(1+g)^{t-1} . noi_1}{(1+r)^t} = \frac{noi_1}{1+r} . \sum_{t=1}^{\infty} \frac{(1+g)^{t-1}}{(1+r)^{t-1}}$$

We also know that any geometric sequence of reason x, with x less than 1, satisfies the following equality:

$$(10)\ \sum_{t=0}^{\infty} x^t = \frac{1}{1-x}$$

If I apply (10) to (9) with $x = \frac{(1+g)}{(1+r)}$:

$$(11)\ P_0 = \frac{noi_1}{1+r} . \frac{1}{1 - \frac{(1+g)}{(1+r)}} = \frac{noi_1}{1+r} . \frac{1}{\frac{(1+r)-(1+g)}{(1+r)}}$$

So to simplify:

$$(12)\ P_0 = \frac{noi_1}{r-g}$$

This can also be written as:

$$(13)\ \frac{noi_1}{P_0} = ir = r - g$$

By definition, the discount rate should be equal to the risk-free rate plus the risk premium relevant for the product or activity concerned. The discount rate should therefore be equal to what we call the expected total return (see Chronicle No. 1). In fact, there are very strong links between several related concepts that we will look at later: the discount rate, the total return, the required return, the IRR, etc.

While the Gordon-Shapiro formula is simple to write and use, we must not lose sight of its limitations, which stem from the assumptions we have had to make.

The first, and most obvious, is that the growth rate of the net operating income must be constant.

The second, less obvious but with important consequences, results from the fact that the simplification carried out by Gordon-Shapiro is only possible for an infinite time horizon...

In the next Chronicles, we will examine the consequences and credibility of these two hypotheses.

These chronicles are linked to my activity at the IEIF, a Paris based think tank on real estate where I conduct research into the modelling of major property variables.
For those less familiar with property analysis, these chronicles can be a source of information and a knowledge base. For experts in the field, their purpose is to launch discussions and exchanges on the various subjects I cover.
Some of the chronicles will be based on known and familiar elements, while others will deal with research elements and present some of the results of my work.