

## CHRONICLE N°9

### The Gordon-Shapiro model for real estate (4/4)

We showed in Chronicle 6 that the Gordon-Shapiro's formula allows us to write the total return ( $tr$ ) as the sum of the initial net income return ( $ir_1$ ) plus the growth rate of net operating income ( $g$ ), assuming that this rate is strictly constant over time and that we have an infinite investment horizon. We have also shown that the initial price ( $P_0$ ) is equal to the initial net operating income ( $noi_1$ ) divided by the discount rate ( $r$ ) minus the growth rate of net operating income:

$$(1) \quad ir_1 = tr - g \Leftrightarrow tr = ir_1 + g \Leftrightarrow P_0 = \frac{noi_1}{r - g}$$

#### Limitations of the Gordon-Shapiro real estate model

We showed in Chronicle 7 that the simplifying assumption of constant growth in net operating income ( $g$ ) is not at all neutral from the point of view of  $P_0$ .

Indeed, if I replace the series of effective growth rates in net rental income for an asset or portfolio by its geometric mean over the period analysed, then the price  $P_0$  obtained by discounting future income changes significantly.

Furthermore, we showed in Chronicle 8 that the infinite horizon assumption actually corresponds to an assumption that the discount rate ( $r$ ) is stable over time. This assumption is also not neutral with regard to the value of  $P_0$ .

In fact, the discount rate ( $r$ ), which must be equal to the expected total return (expected risk-free rate + expected risk premium), cannot be considered stable since 1998 (the starting point for the real data used). And we have shown that if the discount rate changes between the initial purchase period and the resale period, 6-9 years later, then the initial price that would have had to be paid is no longer the same.

## Consistency of the simplified Gordon-Shapiro model with the general definition

If we abandon the reference to capital expenditure (capex), which the Gordon-Shapiro model does not take into account, then Chronicle 1 tells us that:

$$(2) \quad tr = ir + cr = \frac{noi}{Pp} + \frac{\Delta p}{Pp}$$

with:

- $tr$  : total return
- $ir$  : income return
- $cr$  : capital growth
- $noi$  : net operating income
- $\Delta p$  : price variation
- $Pp$  : purchase price including transaction costs

And Chronicle 3 tells us that:

$$(3) \quad cr = \frac{\Delta p}{Pp} = \frac{Sp - Pp}{Pp} = \frac{Sp}{Pp} - 1 = \frac{Snoi}{Pnoi} \cdot \frac{Pir}{Sir} - 1 = \frac{(1 + \partial noi)}{(1 + \partial ir)} - 1$$

with:

- $Sp$  : selling price
- $\partial noi$  : the growth rate of net operating income
- $\partial ir$  : the growth rate of income return

Combining equations (2) and (3) we find **(General formula)**:

$$(4) \quad tr = ir + cr = ir + \frac{(1 + \partial noi)}{(1 + \partial ir)} - 1$$

This can be compared with the Gordon-Shapiro model, which can be written as:

$$(5) \quad tr = ir_1 + g$$

with:  $g$  : the growth rate of net operating income ( $\partial noi$ )

This can be written as follows **(Gordon-Shapiro model)**:

$$(6) \quad tr = ir_1 + \partial noi = ir_1 + (1 + \partial noi) - 1$$

Under the strong assumptions of stability of the growth rate of net operating income ( $g = \partial noi$ ) and an infinite time horizon, i.e. stability of the total return ( $tr$ ) (see Chronicle 8).

But in equation (6), if the total return ( $tr$ ) and the growth rate of net operating income ( $\partial noi$ ) are constant, then the income return ( $ir$ ) must also be constant. This means that its growth rate must be zero ( $\partial ir=0$ ).

Unsurprisingly, if we apply a zero growth rate to the income return in equation (4) we have perfect equality between the general formula and the Gordon-Shapiro model.

## **How should this result be interpreted?**

In fact, **the Gordon-Shapiro model is an equilibrium model**. Under its strong simplifying assumptions, this model presents us the stationary equilibrium state of the property market, as soon as we know the stable growth rate of net operating income (which, in equilibrium, should merge with the indexation rate of rents) and the discount rate of the property in question (which, in equilibrium, implies the knowledge and stability of the risk-free rate and the risk premium of the property in question).

It is because of this equilibrium model characteristic that the Gordon-Shapiro model is most often used. If I know the 'right' rate of growth in net operating income and the 'right' discount rate, then I know the 'right' income return or, which is the same thing, the 'right' purchase price, and so I can compare it with the price the seller is offering me to find out whether or not I'm getting a 'good' deal.

The first problem with this approach is that even if I knew the average growth rate of net rental income over my investment period (which is already unlikely), we have seen that this is not enough to know the right theoretical value of  $P_0$  (see Chronicle 7). To do that, I need to know the path, the history travelled, or yet to be travelled, by successive net operating income growth rates.

The second problem with this reasoning is that the 'right' risk-free rate and the 'right' risk premium also need to be known and stable... Even if we can imagine knowing them perfectly well at a given moment, their stability is an issue. (see Chronicle 8) We live in an environment where the stability of the risk-free rate is totally illusory and where, even in expectation, the risk premium has every reason to move, albeit moderately, but to move all the same.

**In the end, the Gordon-Shapiro model applied to real estate presents a simplistic vision of reality that has the immense disadvantage, when used to support a decision, of misleading us about the rationality of the decision in question.**

This is why we will continue to study and model the dynamics of real estate variables to the best of our ability, in order to shed as much light as possible on individual choices.

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These chronicles are linked to my activity at the IEIF, a Paris based think tank on real estate where I conduct research into the modelling of major property variables.  
For those less familiar with property analysis, these chronicles can be a source of information and a knowledge base.  
For experts in the field, their purpose is to launch discussions and exchanges on the various subjects I cover.  
Some of the chronicles will be based on known and familiar elements, while others will deal with research elements and present some of the results of my work.